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Research article

Permanence and almost periodic solution of a discrete multispecies Gilpin-Ayala mutualism system

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Abstract

This paper discusses a discrete multispecies Gilpin-Ayala mutualism system. We first achieve the permanence of the system. Assume that the coefficients in the system are almost periodic sequences, we obtain the sufficient conditions for the existence of a unique almost periodic solution which is globally attractive. One example together with numerical simulation indicates the feasibility of the main result.

Keywords: Almost periodic solution; Discrete; Gilpin-Ayala mutualism system; Permanence; Global attractivity

1 Introduction

The mutualism system has been studied by more and more scholars. Topics such as permanence, global attractivity and global stability of continuous and discrete mutualism system were extensively investigated(see [1–13] and the references cited therein). Recently, as far as the discrete multispecies Lotka-Volterra ecosystem is concerned(see [12, 14–20] and the references cited therein). Chen [15] studied the dynamic behavior of the discrete $n+m$-species Lotka-Volerra competition predator-prey systems

$$x_i(k+1) = x_i(k)\exp\left[b_i(k) - \sum_{l=1}^{n} a_{il}(k)x_l(k) - \sum_{l=1}^{m} c_{il}(k)y_l(k)\right], \quad i = 1, 2, \cdots, n,$$
$$y_j(k+1) = y_j(k)\exp\left[-r_j(k) + \sum_{l=1}^{n} d_{jl}(k)x_l(k) - \sum_{l=1}^{m} e_{jl}(k)y_l(k)\right], \quad j = 1, 2, \cdots, m.$$

Sufficient conditions which ensure the permanence and the global stability of the systems are obtained; for periodic case, sufficient conditions which ensure the existence of a globally stable positive periodic solution of the systems are obtained.

Notice that the investigation of almost periodic solutions for difference equations is one of most important topics in the qualitative theory of difference equations due to its applications in biology, ecology, neural network, and so forth(see [6, 11–14, 21–27] and the references cited therein). Liao and Zhang [11] studied a discrete mutualism model with variable delays of the forms

$$\begin{cases} N_1(n+1) = N_1(n)\exp\left\{r_1(n)\left[\dfrac{K_1(n) + \alpha_1(n)N_2(n-\mu_2(n))}{1 + N_2(n-\mu_2(n))} - N_1(n-\nu_1(n))\right]\right\}, \\ N_2(n+1) = N_2(n)\exp\left\{r_2(n)\left[\dfrac{K_2(n) + \alpha_2(n)N_1(n-\mu_1(n))}{1 + N_1(n-\mu_1(n))} - N_2(n-\nu_2(n))\right]\right\}. \end{cases}$$

By means of an almost periodic functional hull theory, sufficient conditions are established for the existence and uniqueness of globally attractive almost periodic solution to the system.

Motivated by above, in this paper, we are concerned with the following discrete multispecies Gilpin-Ayala mutualism system

$$x_i(k+1) = x_i(k)\exp\left\{a_i(k) - b_i(k)(x_i(k))^{\theta_{ii}} + \sum_{j=1, j\neq i}^{n} c_{ij}(k)\frac{(x_j(k))^{\theta_{ij}}}{d_{ij} + (x_j(k))^{\theta_{ij}}}\right\}, \tag{1.1}$$

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where $i = 1, 2, \cdots, n$; $x_i(k)$ stand for the densities of species x_i at the kth generation, $a_i(k)$ represent the natural growth rates of species x_i at the kth generation, $b_i(k)$ are the intraspecific effects of the kth generation of species x_i on own population, and $c_{ij}(k)$ measure the interspecific mutualism effects of the kth generation of species x_j on species $x_i (i, j = 1, 2, \cdots, n, i \neq j)$, d_{ij} are positive control constants. θ_{ii} and θ_{ij} are positive constants.

Denote as Z and Z^+ the set of integers and the set of nonnegative integers, respectively. For any bounded sequence $\{g(n)\}$ defined on Z, define $g^u = \sup_{n\in Z} g(n), g^l = \inf_{n\in Z} g(n)$.

Throughout this paper, we assume that:

(H1) $\{a_i(k)\}, \{b_i(k)\}$ and $\{c_{ij}(k)\}$ are bounded nonnegative almost periodic sequences such that

$$0 < a_i^l \leq a_i(k) \leq a_i^u, \quad 0 < b_i^l \leq b_i(k) \leq b_i^u, \quad 0 < c_{ij}^l \leq c_{ij}(k) \leq c_{ij}^u,$$

From the point of view of biology, in the sequel, we assume that $\mathbf{x}(0) = (x_1(0), x_2(0), \cdots, x_n(0)) > \mathbf{0}$. Then it is easy to see that, for given $\mathbf{x}(0) > \mathbf{0}$, the system (1.1) has a positive sequence solution $\mathbf{x}(k) = (x_1(k), x_2(k), \cdots, x_n(k)) (k \in Z^+)$ passing through $\mathbf{x}(0)$.

The remaining part of this paper is organized as follows: In Section 2, we will introduce some definitions and several useful lemmas. In the next section, we establish the permanence of system (1.1). Then, in Section 4, we establish sufficient conditions to ensure the existence of a unique positive almost periodic solution which is globally attractive. The main results are illustrated by an example with numerical simulation in Section 5. Finally, the conclusion ends with brief remarks in the last section.

2 Preliminaries

Firstly, we give the definitions of the terminologies involved.

Definition 2.1([28]) A sequence $x : Z \to R$ is called an almost periodic sequence if the ε-translation set of x

$$E\{\varepsilon, x\} = \{\tau \in Z : \mid x(n+\tau) - x(n) \mid < \varepsilon, \forall n \in Z\}$$

is a relatively dense set in Z for all $\varepsilon > 0$; that is, for any given $\varepsilon > 0$, there exists an integer $l(\varepsilon) > 0$ such that each interval of length $l(\varepsilon)$ contains an integer $\tau \in E\{\varepsilon, x\}$ with

$$\mid x(n+\tau) - x(n) \mid < \varepsilon, \quad \forall n \in Z.$$

τ is called an ε-translation number of $x(n)$.

Definition 2.2([29]) A sequence $x : Z^+ \to R$ is called an asymptotically almost periodic sequence if

$$x(n) = p(n) + q(n), \quad \forall n \in Z^+,$$

where $p(n)$ is an almost periodic sequence and $\lim_{n\to\infty} q(n) = 0$.

Definition 2.3([30]) A solution $(x_1(k), x_2(k), \cdots, x_n(k))$ of system (1.1) is said to be globally attractive if for any other solution $(x_1^*(k), x_2^*(k), \cdots, x_n^*(k))$ of system (1.1), we have

$$\lim_{k\to+\infty} (x_i^*(k) - x_i(k)) = 0, \ i = 1, 2, \cdots, n.$$

Lemma 2.1([31]) If $\{x(n)\}$ is an almost periodic sequence, then $\{x(n)\}$ is bounded.

Lemma 2.2([32]) $\{x(n)\}$ is an almost periodic sequence if and only if, for any sequence $m_i \subset Z$, there exists a subsequence $\{m_{i_k}\} \subset \{m_i\}$ such that the sequence $\{x(n + m_{i_k})\}$ converges uniformly for all $n \in Z$ as $k \to \infty$. Furthermore, the limit sequence is also an almost periodic sequence.

Lemma 2.3([33]) Assume that sequence $\{x(n)\}$ satisfies $x(n) > 0$ and

$$x(n+1) \leq x(n) \exp\{a(n) - b(n)x^\alpha(n)\} \tag{2.1}$$

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for $n \in N$, where $a(n)$ and $b(n)$ are non-negative sequences bounded above and below by positive constants, α is a positive constant. Then

$$\limsup_{n\to+\infty} x(n) \le \left(\frac{1}{\alpha b^l}\right)^{\frac{1}{\alpha}} \exp\{a^u - \frac{1}{\alpha}\}. \tag{2.2}$$

Lemma 2.4([33]) Assume that sequence $\{x(n)\}$ satisfies

$$x(n+1) \ge x(n)\exp\{a(n) - b(n)x^{\alpha}(n)\}, \quad n \ge N_0,$$

$\limsup\limits_{n\to+\infty} x(k) \le x^*$ and $x(N_0) > 0$, where $a(n)$ and $b(n)$ are non-negative sequences bounded above and below by positive constants, α is a positive constant and $N_0 \in N$. Then

$$\liminf_{n\to+\infty} x(n) \ge \left(\frac{a^l}{b^u}\right)^{\frac{1}{\alpha}} \exp\{a^l - b^u(x^*)^{\alpha}\}. \tag{2.3}$$

3 Permanence

In this section, we establish the permanence result for system (1.1).

Proposition 3.1 Assume that (H1) holds. Then any positive solution $(x_1(k), x_2(k), \cdots, x_n(k))$ of system (1.1) satisfies

$$m_i \le \liminf_{k\to+\infty} x_i(k) \le \limsup_{k\to+\infty} x_i(k) \le M_i, \tag{3.1}$$

where

$$M_i = \left(\frac{1}{\theta_{ii} b_i^l}\right)^{\frac{1}{\theta_{ii}}} \exp\left\{a_i^u + \sum_{j=1, j\ne i}^{n} c_{ij}^u - \frac{1}{\theta_{ii}}\right\},$$

$$m_i = \left(\frac{a^l}{b^u}\right)^{\frac{1}{\theta_{ii}}} \exp\{a^l - b^u(M_i)^{\theta_{ii}}\},$$

$i = 1, 2, \cdots, n$.

Proof. From the equations of system (1.1), we have

$$x_i(k)\exp\left\{a_i(k) - b_i(k)(x_i(k))^{\theta_{ii}}\right\} \le x_i(k+1) \le x_i(k)\exp\left\{a_i^u + \sum_{j=1, j\ne i}^{n} c_{ij}^u - b_i(k)(x_i(k))^{\theta_{ii}}\right\}.$$

As the direct conclusion of Lemma 2.3 and 2.4, the inequality (3.1) is completed.

Theorem 3.2 Assume that (H1) holds, then system (1.1) is permanent.

Proposition 3.3 Assume that (H1) holds. Then $\Omega \ne \Phi$.

Proof. By the almost periodicity of $\{a_i(k)\}, \{b_i(k)\}$ and $\{c_{ij}(k)\}$, there exists an integer valued sequence $\{\delta_p\}$ with $\delta_p \to \infty$ as $p \to \infty$ such that

$$a_i(k+\delta_p) \to a_i(k), \quad b_i(k+\delta_p) \to b_i(k), \quad c_{ij}(k+\delta_p) \to c_{ij}(k), \quad \text{as} \quad p \to +\infty.$$

Let ε be an arbitrary small positive number. It follows from Theorem 3.1 that there exists a positive integer N_0 such that

$$m_i - \varepsilon \le x_i(k) \le M_i + \varepsilon, \quad k > N_0.$$

Write $x_{ip}(k) = x_i(k+\delta_p)$ for $k \ge N_0 - \delta_p$ and $p = 1, 2, \cdots$. For any positive integer q, it is easy to see that there exists a sequence $\{x_{ip}(k) : p \ge q\}$ such that the sequence $\{x_{ip}(k)\}$ has a subsequence, denoted by $\{x_{ip}(k)\}$ again, converging on any finite interval of Z as $p \to \infty$. Thus we have a sequence $\{y_i(k)\}$ such that

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$x_{ip}(k) \to y_i(k)$ for $k \in Z$ as $p \to +\infty$.
This, combining with

$$x_i(k+1+\delta_p) = x_i(k+\delta_p)\exp\Big\{a_i(k+\delta_p) - b_i(k+\delta_p)(x_i(k+\delta_p))^{\theta_{ii}} + \sum_{j=1,j\neq i}^{n} c_{ij}(k+\delta_p)\frac{(x_j(k+\delta_p))^{\theta_{ij}}}{d_{ij}+(x_j(k+\delta_p))^{\theta_{ij}}}\Big\}, i = 1, 2, \cdots, n$$

gives us

$$y_i(k+1) = y_i(k)\exp\Big\{a_i(k) - b_i(k)(y_i(k))^{\theta_{ii}} + \sum_{j=1,j\neq i}^{n} c_{ij}(k)\frac{(y_j(k))^{\theta_{ij}}}{d_{ij}+(y_j(k))^{\theta_{ij}}}\Big\}, i = 1, 2, \cdots, n.$$

We can easily see that $\{y_i(k)\}$ is a solution of system (1.1) and $m_i - \varepsilon \le y_i(k) \le M_i + \varepsilon$ for $k \in Z$. Since ε is an arbitrary small positive number, it follows that $m_i \le y_i(k) \le M_i$ and hence we complete the proof.

4 Almost periodic solution

The main results of this paper concern the global attractivity of almost periodic solution of system (1.1) with condition (H1).

Theorem 4.1 Assume that (H1) and

$$\text{(H2)} \quad \rho_i = \max\{|1-\theta_{ii}b_i^l m_i^{\theta_{ii}}|,\ |1-\theta_{ii}b_i^u M_i^{\theta_{ii}}|\} + \sum_{j=1,j\neq i}^{n}\frac{\theta_{ij}c_{ij}^u M_j^{\theta_{ij}}}{d_{ij}} < 1,\ i = 1, 2, \cdots, n,$$

hold. Then system (1.1) admits a unique almost periodic solution which is globally attractive.

Proof. It follows from Proposition 3.3 that there exists a solution $(x_1(k), x_2(k), \cdots, x_n(k))$ of system (1.1) satisfying $m_i \le x_i(k) \le M_i, k \in Z^+$. Let $\{\delta_k\}$ be any integer valued sequence such that $\delta_k \to +\infty$ as $k \to +\infty$. Using the Mean Value Theorem, for $p \neq q$, we get

$$\begin{aligned} \ln x_i(k+\delta_p) - \ln x_i(k+\delta_q) &= \frac{1}{\theta_{ii}\xi_i(k,p,q)}[(x_i(k+\delta_p))^{\theta_{ii}} - (x_i^{\theta_{ii}}(k+\delta_q))^{\theta_{ii}}], \\ \ln x_i(k+\delta_p) - \ln x_i(k+\delta_q) &= \frac{1}{\eta_i(k,p,q)}[x_i(k+\delta_p) - x_i(k+\delta_q)], \end{aligned} \tag{4.1}$$

where $\xi_i(k,p,q)$ lies between $(x_i(k+\delta_p))^{\theta_{ii}}$ and $(x_i(k+\delta_q))^{\theta_{ii}}$, and $\eta_i(k,p,q)$ lies between $x_i(k+\delta_p)$ and $x_i(k+\delta_q)$. Then

$$\begin{aligned} &|(x_i(k+\delta_p))^{\theta_{ii}} - (x_i(k+\delta_q))^{\theta_{ii}}| \le \theta_{ii}M_i^{\theta_{ii}}|\ln x_i(k+\delta_p) - \ln x_i(k+\delta_q)|, \\ &|x_i(k+\delta_p) - x_i(k+\delta_q)| \le M_i|\ln x_i(k+\delta_p) - \ln x_i(k+\delta_q)|, \quad k \in Z^+. \end{aligned} \tag{4.2}$$

For convenience, we introduce $\varphi_i(k,\delta_p,\delta_q)$ through

$$\varphi_i(k,\delta_p,\delta_q) = |\ln x_i(k+\delta_p) - \ln x_i(k+\delta_q)|,\ k \in Z^+,\ \delta_p > 0,\ \delta_q > 0. \tag{4.3}$$

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Thus

$$\begin{aligned}
&\varphi_i(k+1,\delta_p,\delta_q) = |\ln x_i(k+1+\delta_p) - \ln x_i(k+1+\delta_q)| \\
&= \Big|\ln x_i(k+\delta_p) - \ln x_i(k+\delta_q) \\
&\quad + a_i(k+\delta_p) - b_i(k+\delta_p)(x_i(k+\delta_p))^{\theta_{ii}} + \sum_{j=1}^{n} c_{ij}(k+\delta_p)\frac{(x_j(k+\delta_p))^{\theta_{ij}}}{d_{ij}+(x_j(k+\delta_p))^{\theta_{ij}}} \\
&\quad - a_i(k+\delta_q) + b_i(k+\delta_q)(x_i(k+\delta_q))^{\theta_{ii}} - \sum_{j=1}^{n} c_{ij}(k+\delta_q)\frac{(x_j(k+\delta_q))^{\theta_{ij}}}{d_{ij}+(x_j(k+\delta_q))^{\theta_{ij}}}\Big| \\
&\le \Big|\ln x_i(k+\delta_p) - \ln x_i(k+\delta_q) - b_i(k+\delta_p)[(x_i(k+\delta_p))^{\theta_{ii}} - (x_i(k+\delta_q))^{\theta_{ii}}]\Big| \\
&\quad + \Big|a_i(k+\delta_p) - a_i(k+\delta_q)\Big| + \Big|[b_i(k+\delta_q) - b_i(k+\delta_p)](x_i(k+\delta_q))^{\theta_{ii}}\Big| \\
&\quad + \sum_{j=1,j\ne i}^{n}\Big|c_{ij}(k+\delta_p)\big[\frac{(x_j(k+\delta_p))^{\theta_{ij}}}{d_{ij}+(x_j(k+\delta_p))^{\theta_{ij}}} - \frac{(x_j(k+\delta_q))^{\theta_{ij}}}{d_{ij}+(x_j(k+\delta_q))^{\theta_{ij}}}\big]\Big| \\
&\quad + \sum_{j=1,j\ne i}^{n}\Big|\big[c_{ij}(k+\delta_p) - c_{ij}(k+\delta_q)\big]\frac{(x_j(k+\delta_q))^{\theta_{ij}}}{d_{ij}+(x_j(k+\delta_q))^{\theta_{ij}}}\Big|
\end{aligned}$$

$$\begin{aligned}
&\le \Big|\ln x_i(k+\delta_p) - \ln x_i(k+\delta_q) - b_i(k+\delta_p)[(x_i(k+\delta_p))^{\theta_{ii}} - (x_i(k+\delta_q))^{\theta_{ii}}]\Big| \\
&\quad + \Big|a_i(k+\delta_p) - a_i(k+\delta_q)\Big| + \Big|[b_i(k+\delta_q) - b_i(k+\delta_p)](x_i(k+\delta_q))^{\theta_{ii}}\Big| \\
&\quad + \sum_{j=1,j\ne i}^{n}\Big|\frac{c_{ij}(k+\delta_p)}{d_{ij}}[(x_j(k+\delta_p))^{\theta_{ij}} - (x_j(k+\delta_q))^{\theta_{ij}}]\Big| \\
&\quad + \sum_{j=1,j\ne i}^{n}\Big|c_{ij}(k+\delta_p) - c_{ij}(k+\delta_q)\Big|. \qquad (4.4)
\end{aligned}$$

Let ε_1 be an arbitrary positive number. By the almost periodicity of $\{a_{ij}(k)\}$ and $\{b_i(k)\}$ and the boundedness of $\{(x_1(k), x_2(k), \cdots, x_n(k))\}$, it follows from Lemmas 2.2 and 2.4 that there exists a positive integer $K_1 = K_1(\varepsilon_1)$ such that, for any $\delta_q \ge \delta_p \ge K_1$ and $k \in Z^+$(if necessary, we can choose subsequences of $\{\delta_p\}$ and $\{\delta_q\}$),

$$\begin{aligned}
&\Big|a_i(k+\delta_p) - a_i(k+\delta_q)\Big| < \frac{\varepsilon_1}{3}, \\
&\Big|[b_i(k+\delta_q) - b_i(k+\delta_p)](x_i(k+\delta_q))^{\theta_{ii}}\Big| < \frac{\varepsilon_1}{3}, \\
&\sum_{j=1,j\ne i}^{n}\Big|c_{ij}(k+\delta_p) - c_{ij}(k+\delta_q)\Big| < \frac{\varepsilon_1}{3}. \qquad (4.5)
\end{aligned}$$

It follows from (4.1) and (4.3)-(4.5) that, for $k \in Z^+$ and $\delta_q \ge \delta_p \ge K_1$,

$$\begin{aligned}
\varphi_i(k+1,\delta_p,\delta_q) &< \Big|1 - \theta_{ii}b_i(k+\delta_p)\xi_i(k,p,q)\Big|\varphi_i(k,\delta_p,\delta_q) \\
&\quad + \sum_{j=1,j\ne i}^{n}\Big|\frac{\theta_{ij}c_{ij}(k+\delta_p)\xi_{ij}(k,p,q)}{d_{ij}}\Big|\varphi_j(k,\delta_p,\delta_q) + \varepsilon_1 \\
&\le \rho_i \max\{\varphi_i(k,\delta_p,\delta_q)\} + \varepsilon_1,
\end{aligned}$$

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where $\xi_{ij}(k,p,q)$ lies between $(x_i(k+\delta_p))^{\theta_{ij}}$ and $(x_i(k+\delta_q))^{\theta_{ij}}$. Then

$$\begin{aligned}
&\varphi_i(k,\delta_p,\delta_q) < \rho_i \max\{\varphi_i(k-1,\delta_p,\delta_q)\} + \varepsilon_1,\\
&\varphi_i(k-1,\delta_p,\delta_q) < \rho_i \max\{\varphi_i(k-2,\delta_p,\delta_q)\} + \varepsilon_1,\\
&\cdots\cdots\cdots\cdots,\\
&\varphi_i(1,\delta_p,\delta_q) < \rho_i \max\{\varphi_i(0,\delta_p,\delta_q)\} + \varepsilon_1.
\end{aligned}$$

And we have

$$\varphi_i(k,\delta_p,\delta_q) < \rho_i^k \max\{\varphi_i(0,\delta_p,\delta_q)\} + \frac{1-\rho_i^k}{1-\rho_i}\varepsilon_1,$$

for $k \in Z^+$ and $\delta_q \geq \delta_q \geq K_1$.

Since $\rho_i < 1$, for arbitrary $\varepsilon > 0$, there exists a positive integer $K = K(\varepsilon) > K_1$ such that, for any $\delta_q \geq \delta_p \geq K$,

$$\varphi_i(k,\delta_p,\delta_q) < \frac{\varepsilon}{\max\limits_{1\leq i\leq n}\{M_i\}}$$

for $k \in Z^+$.

This combined with (4.2) gives us

$$\Big|x_i(k+\delta_p) - x_i(k+\delta_q)\Big| < \varepsilon \quad \text{for } k \in Z^+ \text{ and } \delta_q \geq \delta_q \geq K.$$

It follows from Lemma 2.3 that the sequence $\{x_i(k)\}(i=1,2,\cdots,n)$ is asymptotically almost periodic. Thus we can express $\{x_i(k)\}$ as

$$x_i(k) = p_i(k) + q_i(k), \tag{4.6}$$

where $\{p_i(k)\}$ are almost periodic in $k \in Z$ and $q_i(k) \to 0$ as $k \to \infty$. In the following we show that $\{p_i(k)\}(i=1,2,\cdots,n)$ ia an almost periodic solution of system (1.1).

Define

$$f_i(k) = a_i(k) - b_i(k)(p_i(k)+q_i(k))^{\theta_{ii}} + \sum_{j=1,j\neq i}^{n} c_{ij}(k)\frac{(p_j(k)+q_j(k))^{\theta_{ij}}}{d_{ij}+(p_j(k)+q_j(k))^{\theta_{ij}}}$$

and

$$g_i(k) = a_i(k) - b_i(k)(p_i(k))^{\theta_{ii}} + \sum_{j=1,j\neq i}^{n} c_{ij}(k)\frac{(p_j(k))^{\theta_{ij}}}{d_{ij}+(p_j(k))^{\theta_{ij}}}, \quad i=1,2,\cdots,n.$$

It follows from (1.1), (4.6) and the Mean Value Theorem that

$$\begin{aligned}
&p_i(k+1)+q_i(k+1)\\
&= [p_i(k)+q_i(k)]\exp\{f_i(k)\}\\
&= p_i(k)[\exp\{f_i(k)\} - \exp\{g_i(k)\}] + p_i(k)\exp\{g_i(k)\} + q_i(k)\exp\{f_i(k)\}\\
&= -p_i(k)\exp\{\xi_i(k)\}\Bigg[b_i(k)[(p_i(k)+q_i(k))^{\theta_{ii}} - (p_i(k))^{\theta_{ii}}]\\
&\quad + \sum_{j=1,j\neq i}^{n} c_{ij}(k)\Big(\frac{(p_j(k))^{\theta_{ij}}}{d_{ij}+(p_j(k))^{\theta_{ij}}} - \frac{(p_j(k)+q_j(k))^{\theta_{ij}}}{d_{ij}+(p_j(k)+q_j(k))^{\theta_{ij}}}\Bigg]\\
&\quad + p_i(k)\exp\{g_i(k)\} + q_i(k)\exp\{f_i(k)\},
\end{aligned}$$

where $\xi_i(k) = \theta_i(k)f_i(k) + (1-\theta_i(k))g_i(k)$ for some $\theta_i(k) \in [0,1]$. Thus

$$\begin{aligned}
& p_i(k+1) - p_i(k)\exp\{g_i(k)\} \\
&= -p_i(k)\exp\{\xi_i(k)\}\Bigg[b_i(k)[(p_i(k)+q_i(k))^{\theta_{ii}} - (p_i(k))^{\theta_{ii}}] \\
&\quad + \sum_{j=1,j\neq i}^{n} c_{ij}(k)\Big(\frac{(p_j(k))^{\theta_{ij}}}{d_{ij}+(p_j(k))^{\theta_{ij}}} - \frac{(p_j(k)+q_j(k))^{\theta_{ij}}}{d_{ij}+(p_j(k)+q_j(k))^{\theta_{ij}}}\Bigg] \\
&\quad - q_i(k+1) + q_i(k)\exp\{f_i(k)\}.
\end{aligned}$$

Let

$$V_i(k) = p_i(k+1) - p_i(k)\exp\{g_i(k)\}.$$

By the boundedness of the almost periodic sequences $\{a_i(k)\}$,$\{b_i(k)\}$,$\{c_{ij}(k)\}$,$\{p_i(k)\}$ and the fact that $q_i(k) \to 0$ as $k \to \infty$, we obtain

$V_i(k) \to 0$ as $k \to \infty$.

We claim that $V_i(k) \equiv 0$. Otherwise, there exists an integer $k_0 \in Z$ such that $V_i(k_0) \neq 0$. By the almost periodicity of $\{a_i(k)\}$,$\{b_i(k)\}$,$\{c_{ij}(k)\}$ and $\{p_i(k)\}$, there exists an integer valued sequence τ_p such that $\tau_p \to \infty$ as $p \to \infty$ and

$$a_i(k+\tau_p) \to a_i(k),\ b_i(k+\tau_p) \to b_i(k),\ c_{ij}(k+\tau_p) \to c_{ij}(k),\ p_i(k+\tau_p) \to p_i(k)$$

uniformly for all $k \in Z$. Then we have

$$\begin{aligned}
V_i(k_0+\tau_p) &= p_i(k_0+\tau_p+1) - p_i(k_0+\tau_p)\exp\{g_i(k_0+\tau_p)\} \\
&\to p_i(k_0+1) - p_i(k_0)\exp\{g_i(k_0)\} \\
&= V_i(k_0)
\end{aligned}$$

as $p \to \infty$, which contradicts that $V_i(k) \to 0$ as $k \to \infty$. This proves the claim. Hence

$$p_i(k+1) = p_i(k)\exp\{g_i(k)\};$$

that is, $\{p_i(k)\}$ is an almost periodic solution of system (1.1).

Assume that $(x_1(k), x_2(k), \cdots, x_n(k))$ is a solution of system (1.1) satisfying (H1). Let

$$x_i(k) = p_i(k)\exp\{u_i(k)\},\quad i = 1,2,\cdots,n.$$

Then system (1.1) is equivalent to

$$\begin{aligned}
u_i(k+1) &= \ln x_i(k+1) - \ln p_i(k+1) \\
&= \ln x_i(k) + a_i(k) - b_i(k)(x_i(k))^{\theta_{ii}} + \sum_{j=1,j\neq i}^{n} c_{ij}(k)\frac{(x_j(k))^{\theta_{ij}}}{d_{ij}+(x_j(k))^{\theta_{ij}}} \\
&\quad - \ln p_i(k) - a_i(k) + b_i(k)(p_i(k))^{\theta_{ii}} - \sum_{j=1,j\neq i}^{n} c_{ij}(k)\frac{(p_j(k))^{\theta_{ij}}}{d_{ij}+(p_j(k))^{\theta_{ij}}} \\
&= u_i(k) - b_i(k)[(x_i(k))^{\theta_{ii}} - (p_i(k))^{\theta_{ii}}] \\
&\quad + \sum_{j=1,j\neq i}^{n} \frac{d_{ij}c_{ij}(k)[(x_j(k))^{\theta_{ij}} - (p_j(k))^{\theta_{ij}}]}{[d_{ij}+(x_j(k))^{\theta_{ij}}][d_{ij}+(p_j(k))^{\theta_{ij}}]} \\
&= u_i(k) - b_i(k)(p_i(k))^{\theta_{ii}}\left[(\exp\{u_i(k)\})^{\theta_{ii}} - 1\right] \\
&\quad + \sum_{j=1,j\neq i}^{n} \frac{d_{ij}c_{ij}(k)(p_j(k))^{\theta_{ij}}[(\exp\{u_j(k)\})^{\theta_{ij}} - 1]}{[d_{ij}+(x_j(k))^{\theta_{ij}}][d_{ij}+(p_j(k))^{\theta_{ij}}]},\quad i = 1,2,\cdots,n.
\end{aligned}$$

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Therefore,

$$\begin{aligned} u_i(k+1) &= u_i(k)\big(1-\theta_{ii}b_i(k)[p_i(k)\exp\{\lambda_i(k)u_i(k)\}]^{\theta_{ii}}\big) \\ &\quad + \sum_{j=1,j\neq i}^{n} \frac{d_{ij}\theta_{ij}c_{ij}(k)u_j(k)[p_j(k)\exp\{\overline{\lambda_j}(k)u_j(k)\}]^{\theta_{ij}}}{[d_{ij}+(x_j(k))^{\theta_{ij}}][d_{ij}+(p_j(k))^{\theta_{ij}}]}, \quad i=1,2,\cdots,n, \end{aligned} \tag{4.7}$$

where $\lambda_i(k), \overline{\lambda_j}(k) \in [0,1]$. To complete the proof, it suffices to show that

$$\lim_{k\to+\infty} u_i(k) = 0, \quad i=1,2,\cdots,n. \tag{4.8}$$

In view of (H2), we can choose $\varepsilon > 0$ such that

$$\rho_i^{\varepsilon} = \max\{|1-\theta_{ii}b_i^l(m_i-\varepsilon)^{\theta_{ii}}|,\ |1-\theta_{ii}b_i^u(M_i+\varepsilon)^{\theta_{ii}}|\} + \sum_{j=1,j\neq i}^{n} \frac{\theta_{ij}c_{ij}^u(M_j+\varepsilon)^{\theta_{ij}}}{d_{ij}} < 1,\ i=1,2,\cdots,n.$$

Let $\rho = \max\{\rho_i^{\varepsilon}\}$, then $\rho < 1$. According to Theorem 3.2, there exists a positive integer $k_0 \in Z^+$ such that

$$m_i - \varepsilon \le x_i(k) \le M_i + \varepsilon, \quad m_i - \varepsilon \le p_i(k) \le M_i + \varepsilon, \quad i=1,2,\cdots,n$$

for $k \ge k_0$.

Notice that $\lambda_i(k) \in [0,1]$ implies that $p_i(k)\exp\{\lambda_i(k)u_i(k)\}$ lies between $p_i(k)$ and $x_i(k)$, $\overline{\lambda_j}(k) \in [0,1]$ implies that $p_j(k)\exp\{\overline{\lambda_j}(k)u_j(k)\}$ lies between $p_j(k)$ and $x_j(k)$. From (4.7), we get

$$\begin{aligned} |u_i(k+1)| &\le \max\{|1-\theta_{ii}b_i^l(m_i-\varepsilon)^{\theta_{ii}}|,\ |1-\theta_{ii}b_i^u(M_i+\varepsilon)^{\theta_{ii}}|\}|u_i(k)| \\ &\quad + \sum_{j=1,j\neq i}^{n} \frac{\theta_{ij}c_{ij}^u(M_j+\varepsilon)^{\theta_{ij}}}{d_{ij}}|u_j(k)|,\ i=1,2,\cdots,n, \end{aligned} \tag{4.9}$$

for $k \ge k_0$.

In view of (4.9), we get

$$\max_{1\le i\le n}|u_i(k+1)| \le \rho \max_{1\le i\le n}|u_i(k)|, \quad k \ge k_0.$$

This implies

$$\max_{1\le i\le n}|u_i(k)| \le \rho^{k-k_0} \max_{1\le i\le n}|u_i(k_0)|, \quad k \ge k_0.$$

Then (4.8) holds and we can obtain

$$\lim_{k\to+\infty}|x_i(k)-p_i(k)| = 0, \quad i=1,2,\cdots,n. \tag{4.10}$$

Now, we show that there is only one positive almost periodic solution of system (1.1). For any two positive almost periodic solutions $p(k) = (p_1(k), p_2(k), \cdots, p_n(k))^T$ and $z(k) = (z_1(k), z_2(k), \cdots, z_n(k))^T$ of system (1.1), we claim that $p_i(k) = z_i(k)(i=1,2,\cdots,n)$ for all $k \in \mathbf{Z}^+$. Otherwise there must be at least one positive integer $K^* \in \mathbf{Z}^+$ such that $p_i(K^*) \neq z_i(K^*)$ for a certain positive integer i, i.e., $\Omega = |p_i(K^*) - z_i(K^*)| > 0$. So we can easily know that

$$\Omega = |\lim_{p\to+\infty} p_i(K^*+\delta_p) - \lim_{p\to+\infty} z_i(K^*+\delta_p)| = \lim_{p\to+\infty}|p_i(K^*+\delta_p) - z_i(K^*+\delta_p)| = \lim_{k\to+\infty}|p_i(k)-z_i(k)| > 0,$$

which is a contradiction to (4.10). Thus $p_i(k) = q_i(k)(i=1,2,\cdots,n)$ holds for $\forall k \in \mathbf{Z}^+$. Therefore, system (1.1) admits a unique almost periodic solution which is globally attractive. This completes the proof of Theorem 4.1. □

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5 Numerical Simulations

Example 5.1 Consider the discrete multispecies Gilpin-Ayala mutualism system:

$$\begin{cases} x_1(k+1) = x_1(k)\exp\Big\{1.26 - 0.02\cos(\sqrt{2}k) - (1.25 - 0.01\sin(\sqrt{3}k))x_1^{\frac{1}{2}}(k) \\ \qquad + (0.025 + 0.002\cos(\sqrt{5}k))\dfrac{x_2^{\frac{1}{3}}(k)}{3 + x_2^{\frac{1}{3}}(k)} + (0.02 + 0.001\cos(\sqrt{2}k))\dfrac{x_3^{\frac{2}{3}}(k)}{1 + x_3^{\frac{2}{3}}(k)}\Big\}, \\ x_2(k+1) = x_2(k)\exp\Big\{1.13 - 0.025\cos(\sqrt{3}k) + (0.02 - 0.003\sin(\sqrt{5}k))\dfrac{x_1(k)}{1 + x_1(k)} \\ \qquad -(1.18 + 0.015\sin(\sqrt{2}k))x_2^{\frac{1}{3}}(k) + (0.025 + 0.002\cos(\sqrt{5}k))\dfrac{x_3^{\frac{3}{2}}(k)}{2 + x_3^{\frac{3}{2}}(k)}\Big\}, \\ x_3(k+1) = x_3(k)\exp\Big\{1.12 - 0.03\cos(\sqrt{3}k) + (0.03 - 0.0025\cos(\sqrt{3}k))\dfrac{x_1^{\frac{1}{2}}(k)}{1 + x_1^{\frac{1}{2}}(k)} \\ \qquad +(0.028 + 0.0015\cos(\sqrt{2}k))\dfrac{x_2(k)}{2 + x_2(k)} - (1.178 + 0.02\sin(\sqrt{5}n))x_3^{\frac{1}{3}}(k)\Big\}. \end{cases} \tag{5.1}$$

A computation shows that

$$\rho_1 \approx 0.0591, \ \rho_2 \approx 0.1004, \ \rho_3 \approx 0.0731,$$

that $\max\{\rho_1, \rho_2, \rho_3\} < 1$. Hence, there exists a unique globally attractive almost periodic solution of system (5.1). Our numerical simulations support our results(see Figs.1,2 and 3).

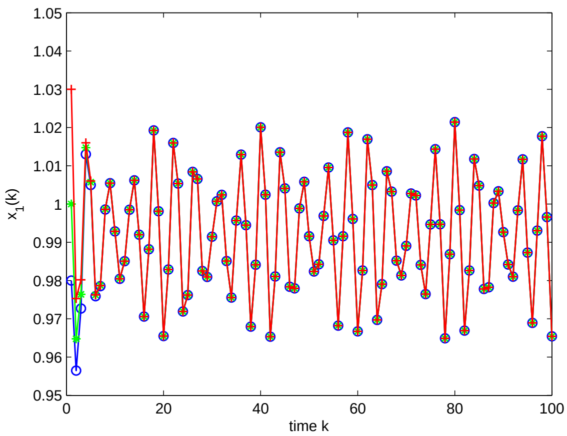

FIGURE1: Dynamic behavior of the first component $x_1(k)$ of the solution $(x_1(k), x_2(k), x_3(k))$ to system (5.1) with the initial conditions (0.98,0.9,0.99), (1,0.94,1.05) and (1.03,0.87,1.1) for $k \in [1, 100]$, respectively.

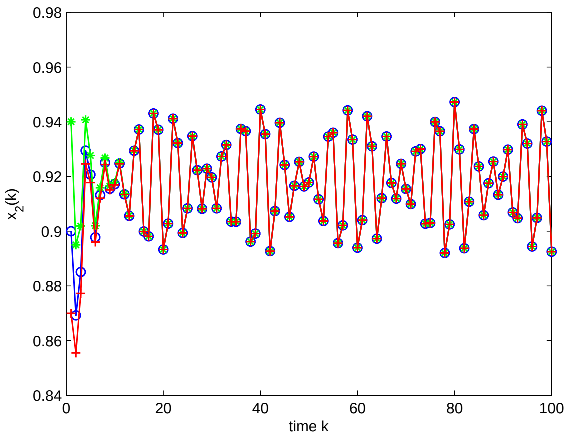

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FIGURE2: Dynamic behavior of the second component $x_2(k)$ of the solution $(x_1(k), x_2(k), x_3(k))$ to system (5.1) with the initial conditions (0.98,0.9,0.99), (1,0.94,1.05) and (1.03,0.87,1.1) for $k \in [1, 100]$, respectively.

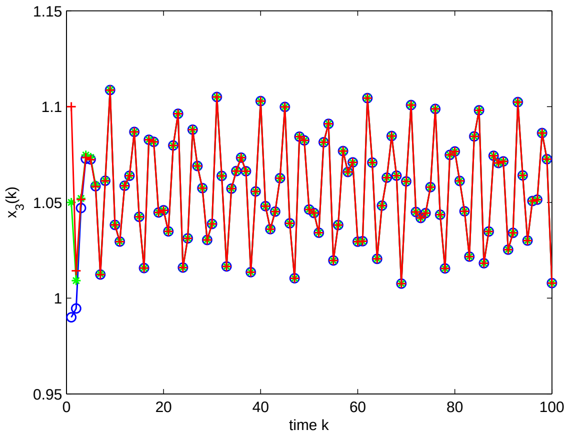

FIGURE3: Dynamic behavior of the third component $x_3(k)$ of the solution $(x_1(k), x_2(k), x_3(k))$ to system (5.1) with the initial conditions (0.98,0.9,0.99), (1,0.94,1.05) and (1.03,0.87,1.1) for $k \in [1, 100]$, respectively.

6 Concluding Remarks

In this paper, assuming that the coefficients in system (1.1) are bounded non-negative almost periodic sequences, we obtain the sufficient conditions for the existence of a unique almost periodic solution which is globally attractive. Furthermore, for the almost periodic discrete multispecies Gilpin-Ayala mutualism system (1.1) with time delays or feedback controls, we would like to mention here the question of how to study the almost periodicity of the system and whether the existence of a unique almost periodic solution is determined by the global attractivity of the system or not. It is, in fact, a very challenging problem, and we leave it for our future work.

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